daughters, while mothers of large litters selectively cannibalize their sons. On average, 2M females mature later than 2F, have rather smaller litters and differ behaviourally in several respects[3,4]. Could the reported differences in sex ratios within day-old litters born to 2M and 2F mothers be due to sex-biased cannibalization of female pups by 2M and of male pups by 2F mothers?

**D. J. Pritchard**
*Department of Human Genetics,*
*University of Newcastle upon Tyne,*
*Newcastle upon Tyne NE2 4AA, UK*

CLARK AND GALEF REPLY — Pritchard notes first, that androgenization of female gerbils gestated between males does not provide a sufficient mechanistic explanation of the phenomenon under discussion and, second, that differences in the sex-bias of maternal cannibalism directed towards large and small litters might explain the observed result. We agree with Pritchard that androgenization of females fails to provide an adequate causal explanation of the effects of intrauterine position on litter sex ratios. We do not, however, believe that our data are consistent with Pritchard's suggestion that sex-biased cannibalism directed towards litters of different sizes can explain our results[1].

Pritchard errs in stating that litters delivered by dams that gestated between two male fetuses (2M females) are rather smaller than are litters delivered by dams that gestated between two female fetuses (2F females). In the paper he cites[3], the difference in mean size of litters delivered vaginally by 32 early- and 36 late-maturing gerbils was not statistically significant. In the 50 litters born to 2M and 2F females studied in ref. 1, the mean size ($\pm$ s.e.m.) of litters delivered by 2M females (6.3 $\pm$ 0.3 pups) did not differ reliably from the mean size of litters delivered by 2F females (6.7 $\pm$ 0.5 pups; Student's $t$ test, $t(48) = 0.68$, NS). It is difficult to see how unreliable differences in litter size could cause reliable differences in litter sex ratio.

To address the more general issue of effects of perinatal mortality on the sex ratios of vaginally delivered gerbil litters, we have compared the sex ratios of caesarian-delivered litters of 2M and 2F female gerbils. To date, caesarian-delivered litters of 2M female gerbils have contained a significantly greater percentage of male fetuses than have caesarian-delivered litters of 2F female gerbils (Mann–Whitney $U$ test, $U = 0$, $P = 0.05$). Although our current sample sizes are

1. Clark, M. M. et al. Nature **364**, 712 (1993).
2. Clark, M. M., Galef, B. G. Jr & Vom Saal.
   Dev Psychobiol. **24**, 81–90 (1991).
3. Clark, M. M. et al. Anim. Behav. **34**, 551–560
   (1986).
4. Clark, M. M. & Galef, B. G. Jr. Physiol. Behav. **42**, 15–18
   (1988).

small ($N$ = three per group), available evidence is not consistent with the hypothesis than any form of sex-biased perinatal mortality is responsible for differences in the sex-ratios of litters delivered vaginally by gerbils gestated in different intrauterine positions.

Because of the scarcity of 2M and 2F females in the relatively small litters produced by Mongolian gerbils, it will be several months before we can complete our examination of sex ratios of caesarian-delivered gerbil litters from 2M and 2F females. However, Vanderbergh and Huggett (personal communications, 19 November 1993) find a highly significant correlation ($P < 0.002$) between the intrauterine positions in which mouse dams (*Mus musculus*) gestate and the sex ratios of their litters. The far greater frequency of 2F and 2M female fetuses in the large litters of mice than in the small litters of gerbils should permit rapid progress in determining the causes of intrauterine-position effects on the sex ratios of rodent litters.

**Mertice M. Clark**
**Bennett G. Galef Jr**
*Department of Psychology,*
*McMaster University,*
*Hamilton, Ontario L8S 4K1, Canada*

# Bayesian quantum mechanics

SIR — Maddox[1] writes of "what many people have been driving at for the past decade, the idea that the probability aspects of quantum mechanics should be built into the foundations of the subject." Wiener and Siegel[2] were early proponents of such a programme. Bohm[3] characterized their ideas as implying "a change of possible structures that would perhaps be as great as that implied by the change from ptolemaic epicycles to newtonian equations of motion."

The differential-space theory used by Wiener and Siegel has been the basis for further work[4-6]. As a consequence of these subsequent analyses, one can assert (speaking in terms of the finite-dimensional case) that an $n \times n$ density matrix ($\rho$) of a quantum system is, in effect, the covariance matrix of a complex multivariate normal (gaussian) distribution over the vectors (wavefunctions) in $n$-dimensional complex Hilbert space. (This can, equivalently, be thought of as a covariance matrix in $2n$-dimensional real space, with a specific form of structure[7,8]. The mean vectors in both the real and complex cases are null.)

There have been few applications of Bayes's theorem[9] in quantum mechanics[10] — though it has been used to prove Bell's theorem[11]. The basic impediment has been the apparent lack of suitable prior

distributions over the pure and mixed states of quantum systems[10]. However — relying upon the multinormal interpretation of $\rho$ — one can use as prior distributions those that have been developed for this class of distributions.

In the bayesian inference of the covariance matrix of a multinormal distribution[12], one can, in the limiting case of minimal prior information[13], attach a vague, Jeffreys, invariant prior, proportional to $|\rho|^{2n+1}$, to $\rho$ (ref. 14). Pure states, for which $|\rho| = 0$, are, thus, assigned null prior probability. In the simplest ($n = 2$, spin-½) case, the improper prior $|\rho|^5$ has been normalized[14], so that its integral over the pure and mixed states is unity. This normalized form is $9,009 (1 - r^2)^5/1,024 \pi$, where $r$ is the distance of a state from the centre in the (Riemann sphere) representation of spin-½ states by the unit ball in three-space. Normalization is more problematical for $n > 2$.

Jones[10] had, in fact, applied Bayes's theorem to the problem of quantum state estimation/determination. However, prior distributions that were simply uniform (unitarily invariant) on the pure states, and null on the mixed states were used. Jones did make and rely on the critical observation that eigenstates onto which measurements project should be regarded as the realizations of the underlying stochastic process. Such realizations could be used to form a sample covariance matrix, from which one could obtain the likelihood and then, by Bayes's theorem, the posterior distribution (the product of the likelihood and the [multinormal] prior $\propto |\rho|^{2n+1}$) (refs 9,11,12).

**Paul B. Slater**
*Community and Organization Research
   Institute,*
*University of California, Santa Barbara,*
*California 93106-2150, USA*

1. Maddox, J. Nature **362**, 693 (1993).
2. Wiener, N. & Siegel, A. Phys. Rev. **91**, 1551–1560 (1953).
3. Bohm, D. Wholeness and the Implicate Order (Routledge and Kegan, London, 1980).
4. Bach, A. J. math. Phys. **21**, 789–793 (1980).
5. Guiasu, S. Phys. Rev. A**36**, 1971–1977 (1987).
6. Levine, R. D. J. stat. Phys. **52**, 1203–1220 (1988).
7. Wooding, R. A. Biometrika **43**, 212–215 (1956).
8. Slater, P. B. J. Phys. A**25**, L359–361 (1992).
9. Howson, C. & Urbach, P. Nature **350**, 371–374 (1991).
10. Jones, K. R. W. Ann. Phys. **207**, 140–170 (1991).
11. Garrett, A. J. M. Found. Phys. **20**, 381–402 (1990).
12. Leonard, T. & Hsu, J. S. J. Ann. Statist. **20**, 1669–1696 (1992).
13. Dickey, J. M., Lindley, D. V. & Press, S. J. Commun. Statist.-Theor. Meth. **14**, 1019–1034 (1985).
14. Slater, P. B. Phys. Lett. A**171**, 285–288 (1992).

---

**Scientific Correspondence**

Scientific Correspondence is intended to provide a forum in which readers may raise points of a scientific character. They need not arise out of anything published in *Nature*. In any case, priority will be given to letters of fewer than 500 words and five references.